A STUDY

OF

ELECTROSTATICALLY COUPLED CIRCUITS

BY

W. Orland Lytle

A thesis submitted to the Department of Physics
and the Faculty of the Graduate School of
the University of Kansas in partial
fulfillment of the requirements
for the Master's Degree.

Laurens E. Whittemore

Department of Physics.

June 5, 1917.

O U T L I N E .

A STUDY OF ELECTROSTATICALLY COUPLED CIRCUITS.

With a view to justifying an extended investigation of electrostatic coupling---if such a justification is needed---let us consider one application which this kind of coupling might have in radio science.

The problem which I was working on when the Great War drew our Nation into the struggle, was that of producing harmonic oscillations in an antenna such as that used for wireless telegraphy. The advantage of such an arrangement is apparent to an operator who desires to use a large antenna for receiving signals and yet who on account of the law or for other reasons must use in transmitting, a wave length which is but a fraction of the natural, or fundamental, wave length of the antenna circuit. If his antenna circuit could be made to oscillate with a wave length which is the first harmonic of the fundamental, he could transmit a wave which would have only one-third the length of the fundamental. If this were not short enough for his purposes, he might use the second harmonic.

Just before the War Department ordered our wireless set dismantled, I was able to produce these harmonics in the antenna which had with the rest of its circuit a fundamental wave length of 665 meters. I had time to try only magnetic coupling between circuits. When the primary circuit of our transmission set is tuned to the same wave length as the fundamental of the antenna circuit we find that the antenna circuit oscillates fundamentally. These statements are true, assuming that the coupling between circuits is not tight enough to allow the secondary to react on the primary, thereby giving two waves, one above and one below the one we expect. When I tuned my primary to a wave length one-third that of the fundamental I got in the antenna this first harmonic with no trace of the fundamental or any other wave length. By reducing the wave length of the primary to one-fifth that of the fundamental I obtained the second harmonic in the antenna.

For the fundamental our primary circuit probably has only enough inductance to give the needed coupling to the antenna circuit, and all the capacity possible, consistent with the reasonable wave length, to keep the energy of the circuit at a high value, having a fixed potential available. Now if we wish to produce the first harmonic, we must reduce the product of inductance and capacity to one-ninth its value for the fundamental. But since our inductance is already no more than we need for our magnetic coupling, it must be the capacity and consequently the energy of our system which is reduced, since $W = 1/2\ CV^2$.

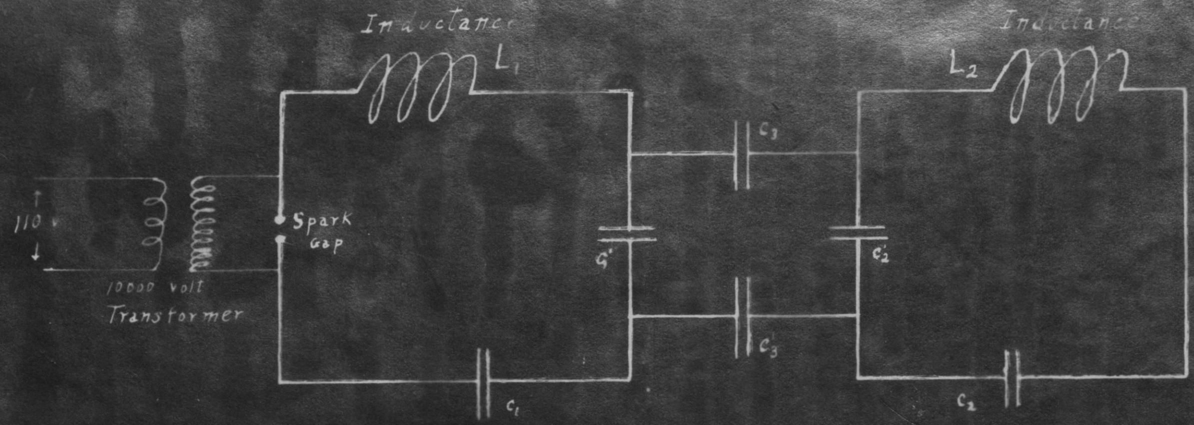


Figure 1.

Electrostatically Coupled Circuits.

Electrostatic coupling between the circuits offers a solution to this problem for we can then reduce the inductance without affecting either the coupling or the energy of the circuit.

So it was that the foregoing work seemed to lead logically to a study of the characteristics of electrostatic coupling. Mr. Laurens E. Whittemore of the Physics Department was just beginning such a study, and so we carried on the work together.

Our purpose in this research was first to investigate the mathematical theory of electrostatically coupled circuits and to test experimentally the truth of the conclusions drawn, then second to study by means of the Braun tube and undamped oscillations the relations existing between the variables in the electrostatically coupled circuits using various values for the coefficient of coupling.

E. Bellini [1] has worked out the mathematical theory of the general case of electrostatically coupled circuits, such as in Fig. 1, by solving the differential equations which may be set up for the circuits from Kirchoff's Laws. Mr. Whittemore took the easier way and solved the equations set up in complex notation form. I will merely

---

[1] La Lumiere Electrique XXXII, 24, 241, 1916.

outline Mr. Whittemore's work.

The equations for potential drops and currents taken in the directions indicated by the arrows for the primary, secondary, and intermediate circuits are the following:

$$i_1\left(jp_1L_1 + \frac{1}{jp_1C_1}\right) + \frac{i_1'}{jp_1C_1'} = 0$$

$$i_2\left(jp_2L_2 + \frac{1}{jp_2C_2}\right) + \frac{i_2'}{jp_2C_2'} = 0$$

$$\frac{i_1'}{jp_1C_1'} + \frac{i_3}{jp_3C_3} + \frac{i_3}{jp_3C_3'} + \frac{i_2'}{jp_2C_2'} = 0$$

$$i_1' = i_1 + i_3$$

$$i_2' = i_2 + i_3$$

Assuming that $p_1 = p_2 = p_3$, equating the determinant to zero, and solving for p^2

$$p'^2 = \frac{1}{2}\left\{\frac{1}{L_1G_1} + \frac{1}{L_2G_2}\right\} - \sqrt{\frac{1}{4}\left\{\frac{1}{L_1G_1} - \frac{1}{L_2G_2}\right\}^2 + \frac{Ne^2}{L_1G_1L_2G_2}}$$

$$p''^2 = \frac{1}{2}\left\{\frac{1}{L_1G} + \frac{1}{L_2G_2}\right\} + \sqrt{\frac{1}{4}\left\{\frac{1}{L_1G_1} - \frac{1}{L_2G_2}\right\}^2 + \frac{Ne^2}{L_1G_1L_2G_2}}$$

where $$\frac{1}{G_1} = \frac{1}{C_1} + \frac{1}{C_1'} - \frac{C_t}{C_1'^2}$$

and $$\frac{1}{G_2} = \frac{1}{C_2} + \frac{1}{C_2'} - \frac{C_t}{C_2'^2}$$

$$Ne^2 = \left\{\frac{Ct}{C_1' C_2'}\right\}^2 G_1 G_2$$ Where Ne is the coefficient of coupling.

$$\frac{1}{C_t} = \frac{1}{C_1'} + \frac{1}{C_2'} + \frac{1}{C_3} + \frac{1}{C_3'}$$

Taking the special case where the frequencies of the two circuits are the same __before__ coupling,

$$\frac{1}{L_1}\left\{\frac{1}{C_1} + \frac{1}{C_1'}\right\} = \frac{1}{L_2}\left\{\frac{1}{C_2} + \frac{1}{C_2'}\right\} = 4\pi^2 n^2$$

where n is the natural frequency of each circuit, from which we get

$$p' = 2\pi n$$

$$p'' = \sqrt{4\pi^2 n^2 - \frac{Ct}{L_1 C_1'^2} - \frac{Ct}{L_2 C_2'^2}}$$

$$\frac{\lambda'}{\lambda''} = \frac{\sqrt{1 - Ne}}{\sqrt{1 + Ne}}$$ where λ' and λ'' are wave lengths.

When the frequencies of the two circuits are the same __after__ coupling, i.e. when each circuit was tuned to the same wave length with the intermediary coupling condensers connected

$$L_1 G_1 = L_2 G_2$$

and we get

$$p' = \sqrt{\frac{1 - Ne}{L_1 G_1}}$$

$$p'' = \sqrt{\frac{1 + Ne}{L_1 G_1}}$$

$$n' = n\sqrt{1 + Ne} \qquad \text{or} \qquad \lambda' = \frac{\lambda}{\sqrt{1 + Ne}}$$

$$n'' = n\sqrt{1 - Ne} \qquad \text{or} \qquad \lambda'' = \frac{\lambda}{\sqrt{1-Ne}}$$

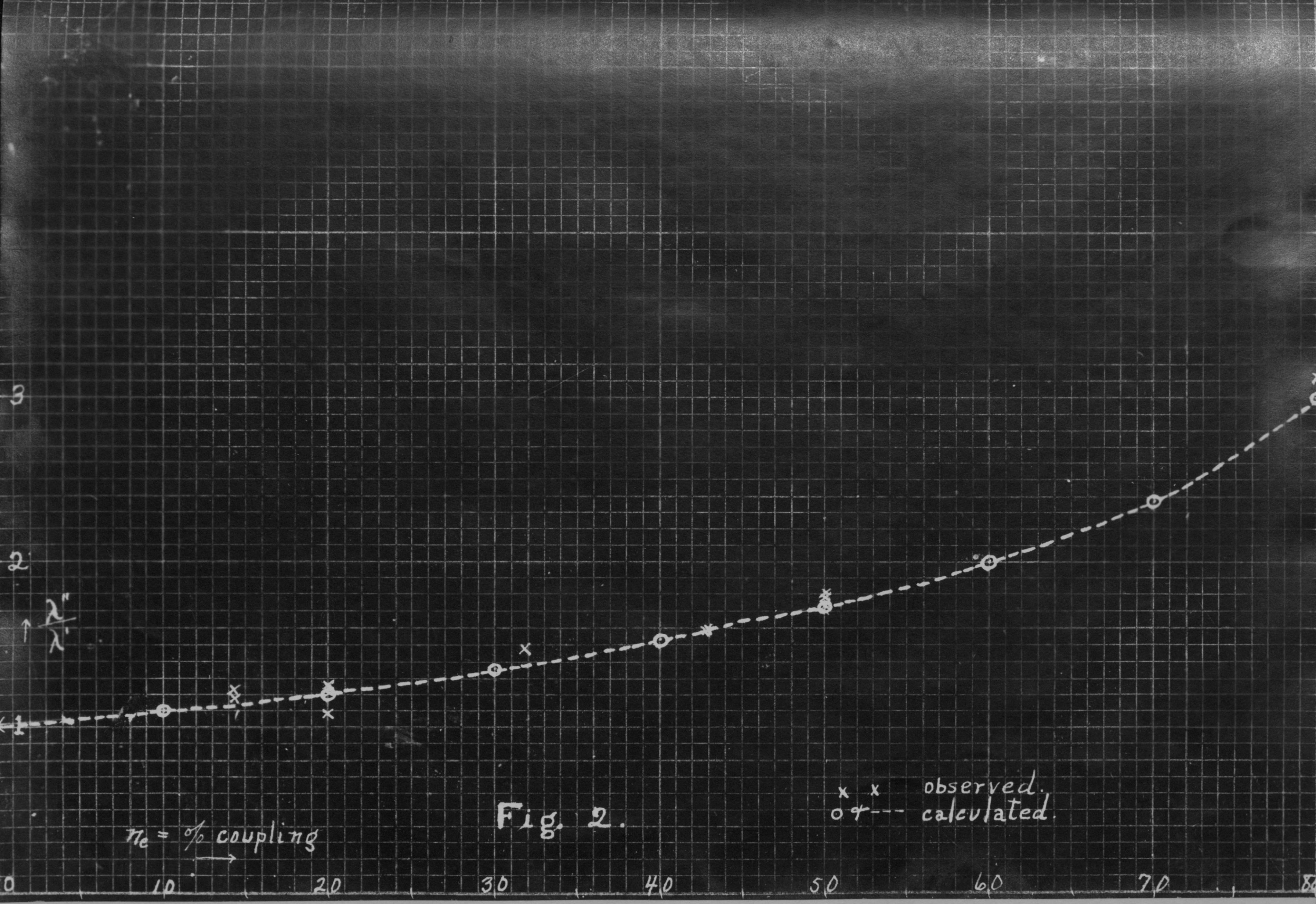
In both cases the shorter wave corresponds to the natural frequency of one of the single circuits without the coupling condenser.

To comply with the condition that each circuit have the same frequency before coupling we excited each circuit separately, when the coupling capacities were not connected, by a spark gap and transformer and tuned each to the desired wave length, as determined by a resonating wave meter. The circuits were then connected electro-statically and the system excited using a spark gap in one circuit. The waves present in each circuit were determined with the wave meter.

For the other condition, we tuned each circuit when the other circuit was broken somewhere not between the coupling connections. The connections were then made and the wave lengths in each circuit determined. The theory was tested for the extremes in values for coupling as well as for a number of intermediate values. We increased capacities conveniently sometimes by merely shorting the condenser.

The following is the data for our observed and calculated values, from which Fig. 2 is plotted:

OBSERVED					CALCULATED			
n_e	λ	λ'	λ''	$\frac{\lambda''}{\lambda'}$	λ'	λ''	n_e	$\frac{\lambda''}{\lambda'}$
.80	---	290	908	3.13	290	872	.0	1.00
.50	---	407	727	1.78			.1	1.10
.50	510	414	722	1.74	417	722	.2	1.22
.00	---	525	525	1.00			.3	1.36
.20	---	285	358	1.26			.4	1.53
.20	315	285	355	1.25			.5	1.73
.32	---	405	597	1.47	405	564	.6	2.00
.43	---	296	465	1.57			.7	2.38
.50	---	300	515	1.71	300	520	.8	3.00
.50	363	291	520	1.79			.9	4.36
.43	355	300	475	1.58	298	470	1.0	∞
.40	450	390	553	1.42	381	582		
.20	342	322	375	1.16				
.20	450	410	495	1.21				
.14	215	200	237	1.18	201	233		
.14	---	198	243	1.22	198	232		
1.00	317	225	∞	∞				
1.00	317	555 278	∞	∞				
1.00	153	555 150	∞	∞				
1.00	218	157	∞	∞				



3
2
1
$\frac{\lambda''}{\lambda'}$
x x observed.
o+--- calculated.
n_c = % coupling
0
10
20
30
40
50
60
70
80

Fig. 2.

The curve (Fig. 2) gives a good comparison of observed and calculated values. We see that as our coefficient of coupling, Ne, approaches closer and closer to unity, one wave approaches infinity in length under both conditions of tuning. When the circuits are in tune after coupling, while the one wave length approaches infinity the other approaches zero. But in the case of tuning before coupling we have our one wave always the same while the other approaches infinite length and no energy content. This is surely an ideal state of things when we can transfer nearly all of the energy of the primary into the secondary and yet have that energy in only one wave.

The Braun tube method of studying the arc phenomena of single circuits is not by any means new, but we are reasonably sure that the effects produced by electrostatically coupling a secondary circuit have never been investigated by this method.

Professor Simon[2] explored the "dynamic characteristic" of an alternating current arc by means of a Braun tube arranged so that the cathode ray pointer was deflected horizontally by the arc current and vertically by the potential difference across the arc. The closed curve he obtains is of the form of Figure 3 and shows a phenomenon

---

[2] H. Th. Simon, "Phys. Zeitschr." VI, p. 297, 1905; VII, p. 423, 1906.

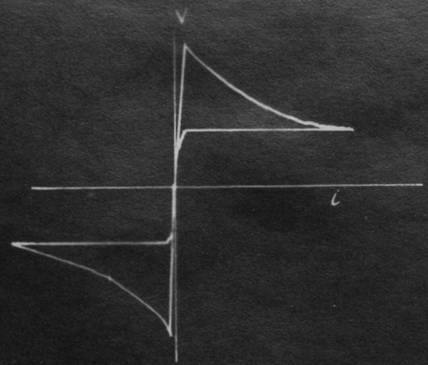


Fig. 3

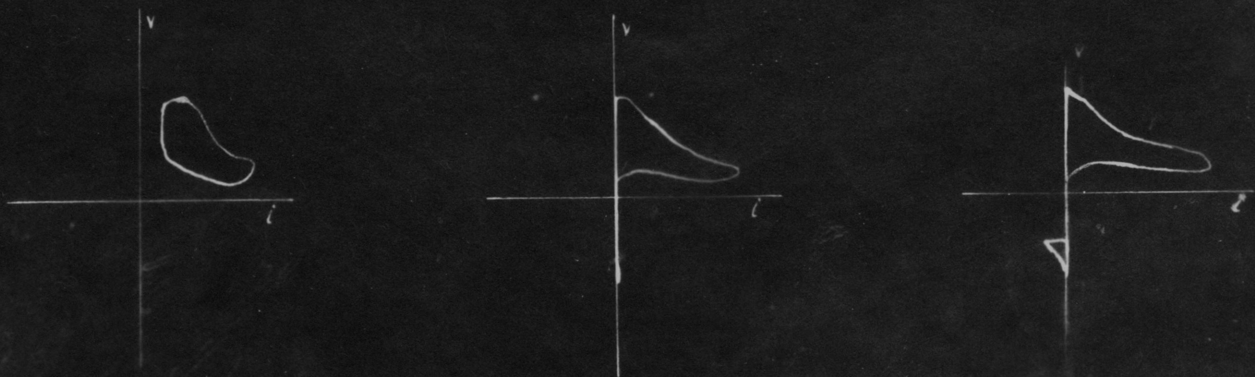


Fig. 4

called "arc hysteresis." This shows very nicely how the variables are related when so that the arc is said to have a falling characteristic. In 1900 Mr. Duddell[3] showed that a D.C. arc gave out a musical note when it was shunted by a condenser and inductance of proper proportions. The most extensive and valuable study of the dynamic characteristics of the oscillating arc was made by Simon and his students.

[4] Mr. Hidetsugu Yagi has investigated the reacting effect of a magnetically coupled secondary circuit upon the oscillation of a carbon arc. In our experiments we are substituting known value of electrostatic coupling for his magnetic coupling.

There are three types of oscillations which may be obtained with an arc. If there were no oscillations the current thru the arc, i_o, would be nearly constant. The condenser discharge thru the arc tends to superpose a sinusoidal current and make the current pulsating. So long as i_o is larger than the amplitude of pulsation, there is no extinction of the arc and the oscillation is said to be of the "first type." The oscillation of this type is generally obtained in musical arcs. When the fluctuation becomes larger than i_o, there will be a duration of no current and the arc will be extinguished for a moment. If the arc extinguishes,

---

(3) W. Duddell, "Journal I.E.E., Vol. 30, p. 232, 1900.
(4) Hidesugu Yagi, Proc. Inst. of Radio Engineers, 4, p. 371, 1916.

a constant current, i_0 , will flow into the condenser and charge it up until its potential becomes sufficiently high to cause the next discharge across the arc gap. This is called the "oscillation of the second type" and is most readily obtained in practice at radio frequencies, especially when there is any dissimilarity of electrode material. If the terminal P. D. which becomes reversed at the extinction is large enough to cause a discharge across the gap, it will light a small arc in the opposite direction. The oscillation with this reverse discharge is of the third type. The three types are diagramatically represented in Fig. 4, as taken from Mr. Yagi's paper.

As the second type of oscillations are used in radio science, we have used this type in our study.

In our experimental work we studied the relations between 1) $\frac{di}{dt}$, and i in the primary, where i is current; 2) P.D. across condenser in secondary, and primary current; 3) i in secondary, and i in primary; 4) P.D. across arc, and secondary i; 5) $\frac{di_2}{dt}$ and i_2; 6) P.D. across arc, and i_1, which however was not very successful because our Braun tube was not constructed so as to give us the necessary amplitude for our potential deflections. Before beginning the above studies in electrostatic coupling we reproduced some of Mr. Yagi's work with magnetic coupling in order to be sure that the apparatus was being used in the proper

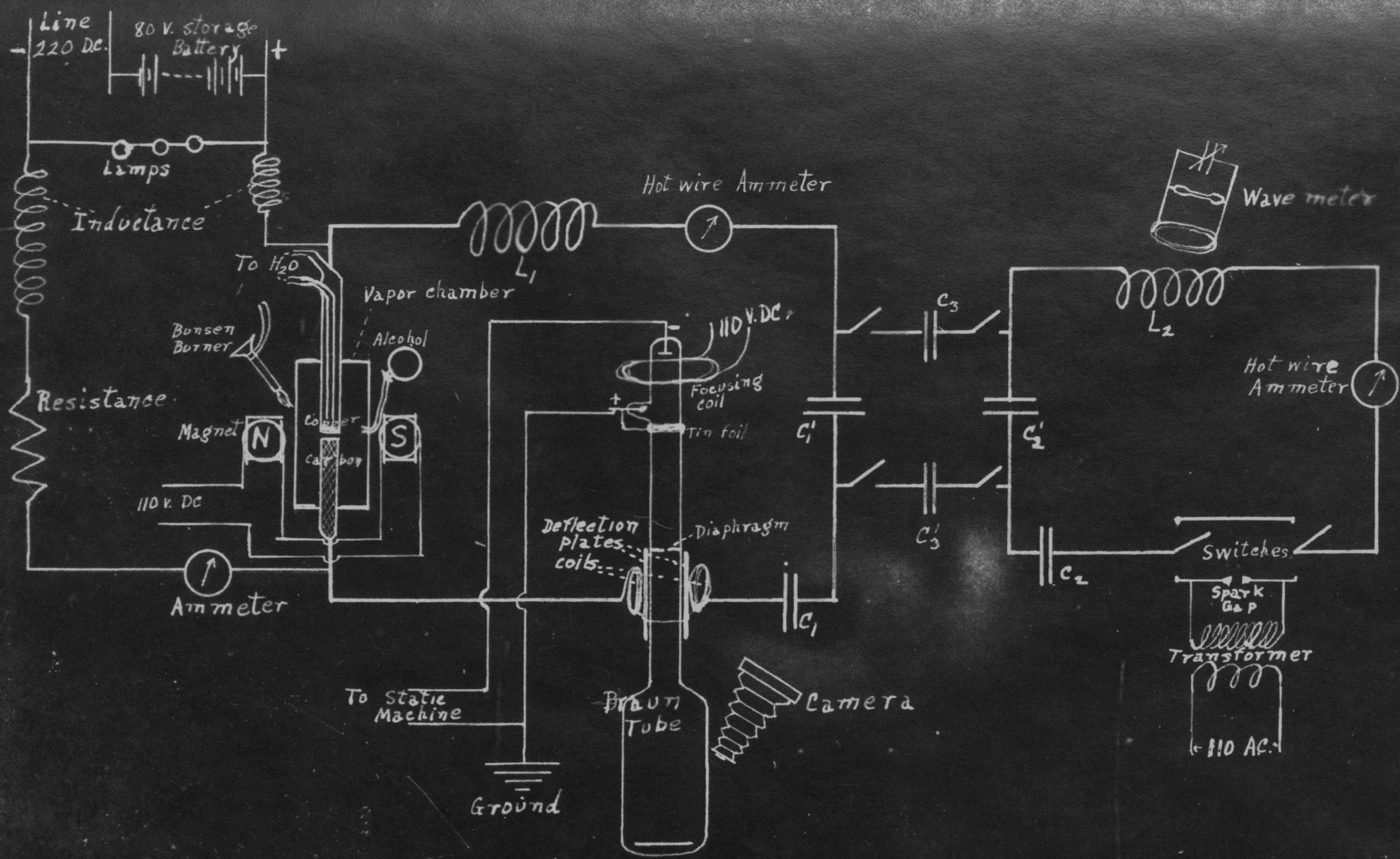

Line
220 DC.
80 V. storage
Battery
Lamps
Inductance
Hot wire Ammeter
Wave meter
L_1
To H_2O
Vapor chamber
Bunsen
Burner
Alcohol
110 V. DC
Focusing
coil
Resistance
Magnet
N
S
Copper
Carbon
Tin foil
C_1'
C_3
L_2
Hot wire
Ammeter
C_2'
110 V. DC
Deflection
plates
coils
Diaphragm
C_3'
Switches
C_2
Spark
Gap
Ammeter
C_1
Transformer
To Static
Machine
Braun
Tube
Camera
110 AC.
Ground

Fig. 5.

way and to accustom ourselves to the manipulations necessary.

Figure 5 shows diagramatically the arrangement of apparatus for our work as used with the various connections. We shall call the circuit shunting the arc the primary circuit. Our first experimental problem was to construct an arc which would give us fairly persistent oscillations in our shunt circuit. They had to be steady enough to produce a figure on the Braun tube screen which could be photographed. After many trials and failures of different arrangements we finally used the enclosed arc shown in the diagram, with fair success. The negative electrode was a solid carbon rod about one centimeter in diameter with its end filed off flat. This last is necessary to keep the arc from varying in length as it moves around. The carbon should be rotated slowly to prevent the arc from consuming one point on the carbon, but this we did not do, as We found it sufficient to turn the carbon part way around once in a while, or to substitute a new carbon. Sometimes we used a longitudinal magnetic field, which caused the arc to revolve about the axis of the carbon. The transverse magnetic field produced more vigorous oscillations but generally not such steady ones as no field at all.

The arc was enclosed in a porous cup which was properly closed with asbestos and provided with a peep hole for the adjustment of the arc and alcohol drip shown in the diagram. A flame from a bunsen burner kept the porous cup hot so that the alcohol which dripped down on the inside was quickly vaporized. The alcohol vapor seemed necessary, for, as soon as the alcohol gave out the oscillations stopped. This alcohol vapor has the effect of steepening the characteristic curve of the arc.

Our Braun tube did not have some features which we wanted but we used it as it was. There was only one diaphragm in the tube when there should have been two to make the spot on the screen small and clearly defined. A focusing coil placed as in the drawing helped us greatly in obtaining a bright and fairly well defined spot. The strength of the field of this coil and its direction had to be adjusted by experiment for the best effect. We were troubled by the jumping of the beam, due probably to the accumulation of charges on various parts of the tube, until we partially covered the large end of the tube with tinfoil and put a strip of foil around the tube close to the positive electrode, grounding each of these foils along with the positive electrode. Current deflections were obtained by passing the current thru two coils placed on opposite sides of the tube so that their magnetic fields would add. In our experiments two coils of twenty turns each were used in series. For potential deflections a couple of metal plates were held as close as possible to the tube and on opposite sides. If our plates had been sealed into the tube we would have been able to get larger deflections for the potentials used---a thing which we often needed. For running the tube we used a two-plate Wimshurst electrostatic machine driven by a motor.

Placed a little to one side of the tube and focused on the front of the screen we had our camera. With an f. 6.3 Anastigmat lens and Cramer's Isochromatic plates an exposure of from one-fifth to three-fifths of a second was sufficient for good photographs. Once in a while we thought our plates showed the effects of x-rays, probably from the aluminum diaphragm in the tube.

The ammeters used in the two oscillatory circuits were of the hot wire type, each being calibrated with the line ammeter and direct current. However, as the secondary current meter was burned out just preceding its calibration, it was calibrated using a wire and shunt as nearly like the original as possible. At any rate, even if the values thereby given are only approximate, we know relative values from the readings.

The inductance in each circuit aside from that in the deflection coils was in the shape of a spiral. The capacity was made up of sections of Murdock molded condensers of approximately .0017 microfarads capacity each. The capacities of all sections were assumed equal when the coefficient of coupling was calculated. A variable air condenser, with a capacity at fifty-five scale divisions equal to that of one section of condenser, was used with the coupling capacity to make the coupling continuously variable.

When the switches shown in the secondary circuit are thrown towards the spark gap we have a means of exciting our secondary for tuning purposes. The wave meter consisted of an inductance in series with a variable air condenser calibrated for wave lengths. A low pressure hydrogen tube was connected across the terminals of the inductance or condenser, to indicate maximum potentials in the circuit.

On the line side of the arc there was some dead resistance for controlling the current thru the arc, as well as a large inductance in each line to prevent oscillations from the shunt circuit from getting into the line. The inductance in each line was the secondary of a commercial

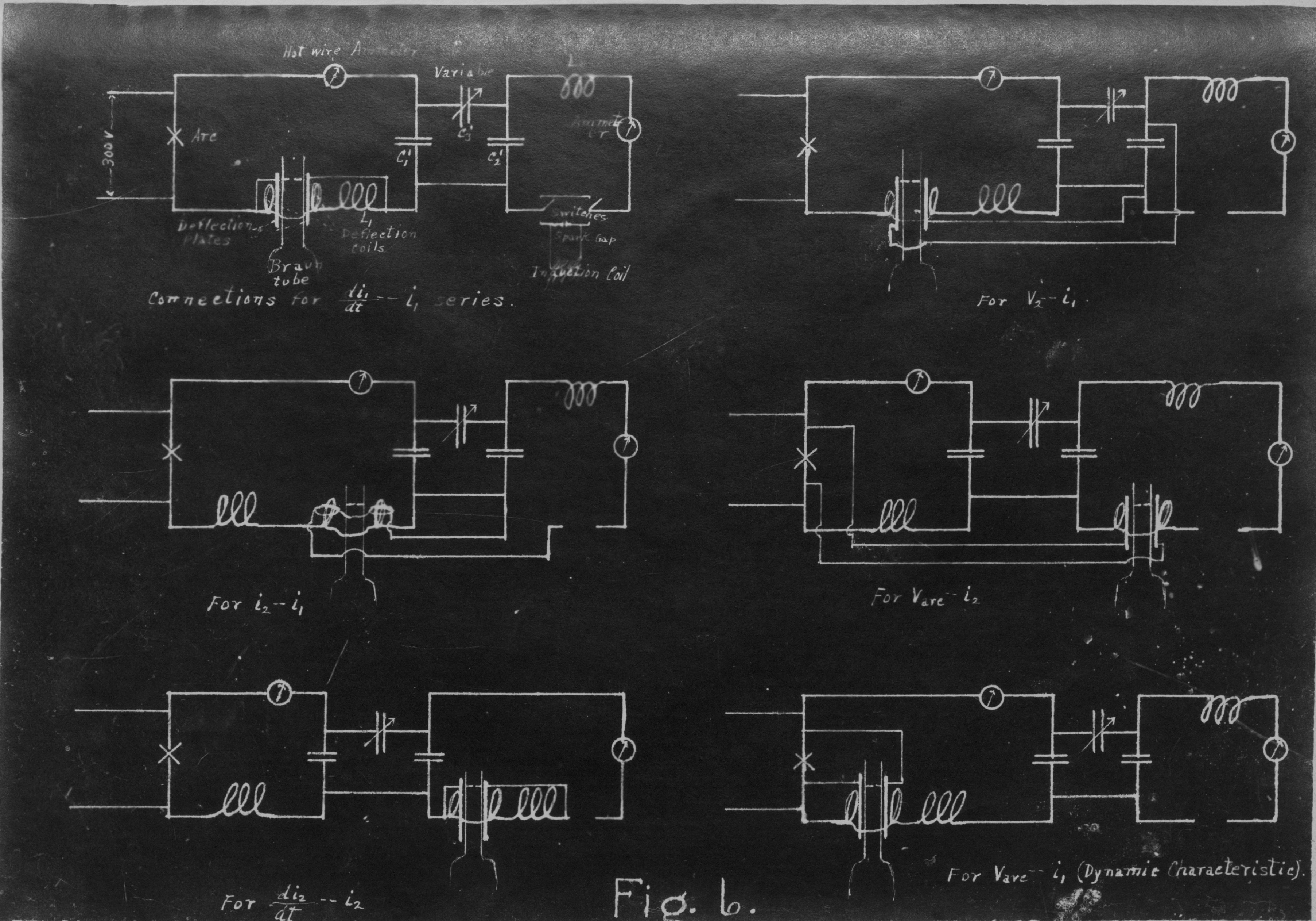

Hot wire Ammeter
Variable
L
Arc
300 V
C_1'
C_3'
C_2'
Ammeter
Deflection plates
L_1
Deflection coils
Switches
Spark Gap
Induction Coil
Braun tube
Connections for $\frac{di_1}{dt}$ -- i_1 series.
For V_2 -- i_1
For i_2 -- i_1
For V_{arc} -- i_2
For $\frac{di_2}{dt}$ -- i_2
For V_{arc} -- i_1 (Dynamic Characteristic).

Fig. 6.

house supply transformer. As an additional precaution against the straying of the oscillations, three incandescent lamps were placed in series across the line.

In our work we found it very convenient to short-circuit C_1, C_3, and C_2, and to use one section of condenser for C_1' and C_2' each, leaving C_3' for varying the coupling. With this arrangement and with all the capacity we had available, the coupling could be varied from zero to over ninety per cent.

In each case we were careful to have our circuits tuned so that they satisfied the condition of resonance before coupling. The wave length of an oscillation in the primary circuit, which seemed to be readily reproduced, was determined and the secondary was tuned to that, without the coupling by means of the spark gap and transformer. Now according to the theory which we have verified in the first part of this work we should always have this original frequency in each of the circuits no matter what the value of the coupling. Therefore after coupling we placed the wave meter near the primary circuit and adjusted the arc until the tube on the meter glowed when the instrument was set for the original wave length.

The same procedure was used in obtaining each series of relations between variables. Having our deflecting coils and plates on the tube properly connected, we began with zero coupling between the circuits and increased to the maximum taking photographs as we proceeded whenever we got a new figure or a great change in a preceding one. As each photograph was taken, we noted the value of the coefficient of coupling and the primary, secondary, and line currents. For the two current deflections we used two sets of coils at right angles on the tube, which had for our purposes practically no mutual inductance, as we found by test.

The following table gives in the rows the series with the same variables while the columns give those figures of the different series with approximately the same coupling. Since a figure generally evolved gradually into the next figure taken in that series we can easily "interpolate" figures to fill out some of our columns, if we care to develop the set of figures with any certain coupling.

CLASSIFICATION OF PHOTOGRAPHS ACCORDING TO COUPLING VALUES.

Numbers in table refer to numbers on photographs.

Coupling %	0	30	33	42	56	61	65	70	74	78	82	87	90	92
(1) $\frac{di_1}{dt} / i_1$	—	—	—	—	13	—	—	—	—	14	15	16	17	18 19
(2) $\frac{V_2}{i_1}$	20	21	22	—	23	—	21	24	—	25	26	—	27	28
(3) $\frac{i_2}{i_1}$	29	—	—	30	31	—	32	—	33	—	34	35	36	37
(4) $\frac{V_{arc}}{i_2}$	38	—	—	39	—	40	41	—	—	—	42 43	—	44	45 46
(5) $\frac{di_2}{dt} / i_2$	47	—	—	—	—	48	49 50	—	51	—	52 53	54 55	56	57 58

In figure 7 we have plotted the current values in the secondary circuit as they varied with the coupling as obtained in series (4). Similar curves were obtained for the other series except in series (1) and (3) where the second maxima seem to be missing. Considering Figure 7, the first maximum of current occurs at a place where the value of coefficient of coupling was sixty percent. Now if the curve (Fig. 2) showing $\frac{\lambda''}{\lambda'}$ for various coupling values, be consulted, it will be seen that for Ne = 60%, $\frac{\lambda''}{\lambda'} = 2$. The second maximum on Fig. 7 is at Ne = 80%, at which value by curve 1, $\frac{\lambda''}{\lambda'} = 3$. By reference to the photographs of the figures obtained at these values the values of $\frac{\lambda''}{\lambda'}$ obtained above are verified.

. Therefore when the ratio of the two frequencies is an integer the root mean square value of the current is a maximum. The variation of the primary current also shown in Fig. 7 leads us to the conclusion as stated for the secondary circuit. Nothing definite can be said about the line current unless it is that it seems to be a minimum when the oscillating current is a maximum.

As series (1) gives us the value of $\frac{di_1}{dt}$ with respect to i_1, it is useful for the exploration of the variation of currents, and consequently potentials, with

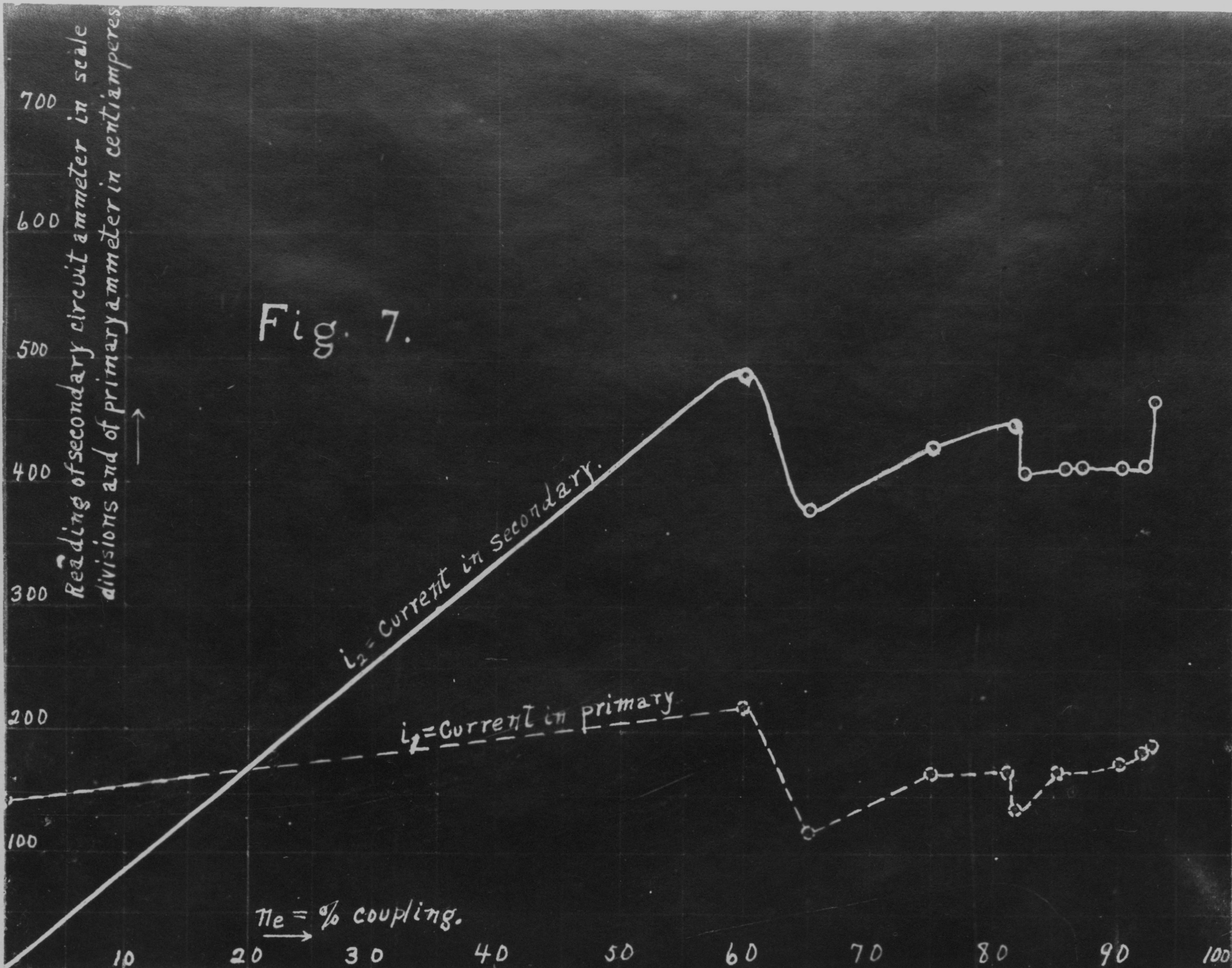


Fig. 7.
Reading of secondary circuit ammeter in scale divisions and of primary ammeter in centiamperes
i_2 = Current in secondary.
i_1 = Current in primary
n_e = % coupling.
700
600
500
400
300
200
100
10
20
30
40
50
60
70
80
90
100

respect to time. Let us take figures which were obtained with 82% coupling and develop the curves for time.

Fig. 8 shows, first the figure as obtained from the photograph properly placed for development, and then the resulting curve. This is not the actual shape of our wave, as we cannot determine equal intervals of time on our figure, but it does tell us something of the number and relative positions of maxima, minima, and constant values. One thing we do know is that the areas per cycle above and below the i_s line must be equal because the quantity of electricity put into the condenser (equal to $\int idt$) is equal to that discharged by it.

Fig. 9 gives the potential across the secondary condenser with as it varies with time. This is obtained from the figure to the left by comparison with the curve obtained in Fig. 8. By the same method Fig. 10 is drawn. As Fig. 10 plots the relation between secondary current and time, we should be able to get the same shaped curve by developing the cyclic diagram of the secondary current and its time rate of change as shown in Figure 11. It will be seen that these figures do agree. Figs 9, 10, and 11 are drawn for only a half cycle or a little more. The relation between P. D. across the arc and secondary current is not

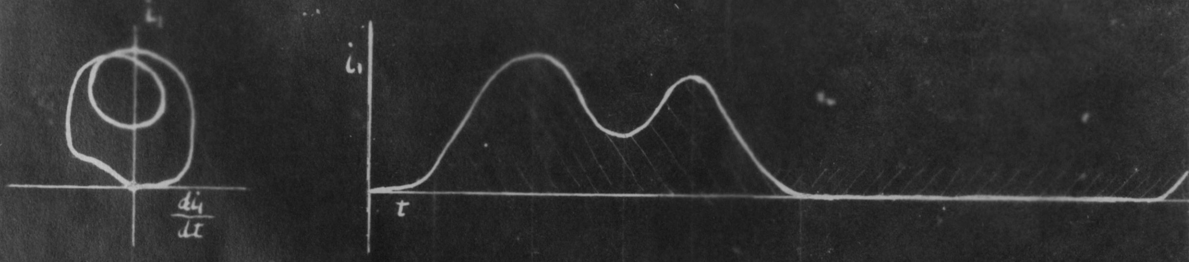


Fig. 8.

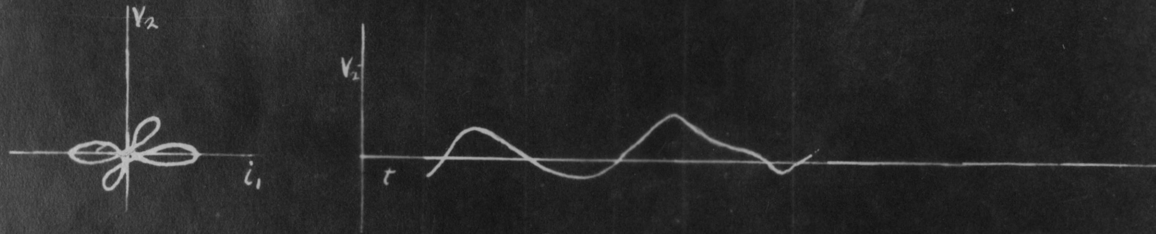


Fig. 9.

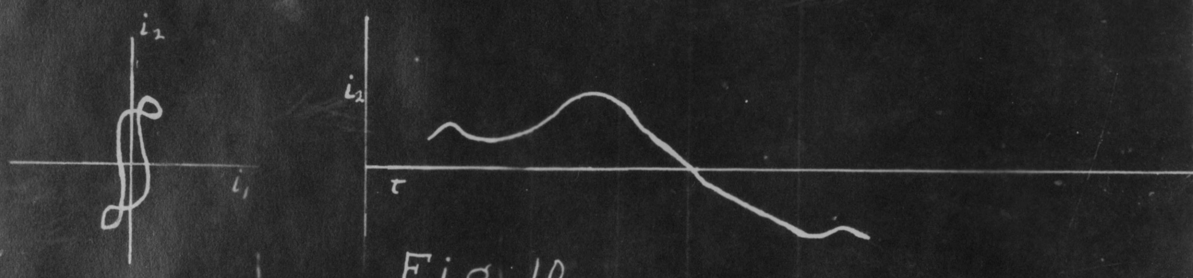


Fig. 10.

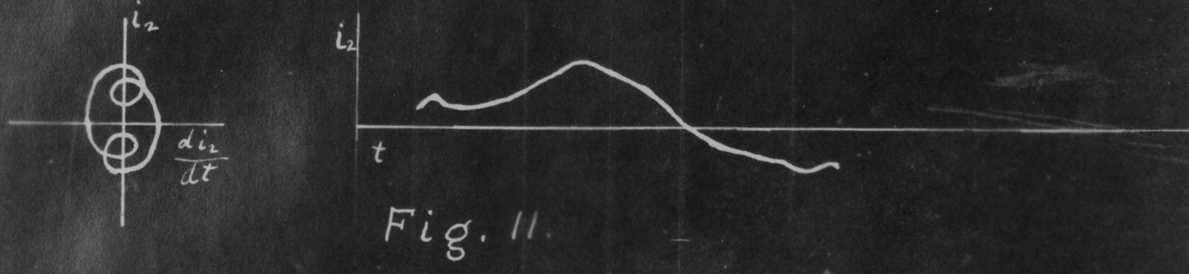


Fig. 11.

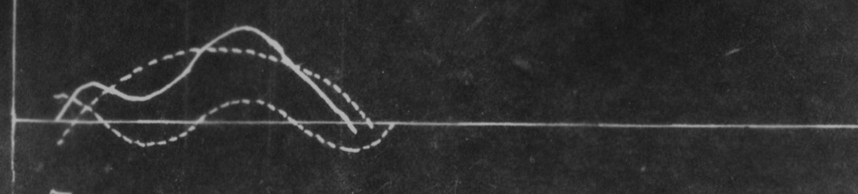


Fig. 12.

developed because of the uncertainty of the path as shown by the photograph. Fig. 12 shows how the addition of two sine curves, one with three times the frequency of the other gives a curve similar to the one obtained in Fig. 10. One the shape of Fig. 8 may be obtained by adding the two curves as above if the phase relations are changed a quarter of a period. This relation between the frequencies is in agreement with the determination made heretofore.

Figures subsequent to Fig. 12 are photographs of the figures produced on the Braun tube screen. The figures of the same series or those with the same coupling value may be picked out by reference to the table on page 17.

From this study one is encouraged to believe that electrostatic coupling should have a place in the transference of energy between radio circuits first, because a high degree of coupling is possible and second, because there is practically only one wave in the circuits when such a high coupling value is used.

I desire to thank Mr. Laurens E. Whittemore of the Physics Department for his constant and untiring direction and help in this work. To the Physics Department also I wish to express my appreciation for the use of apparatus used in this research.

Fig. 13. Fig. 14 Fig. 15. Fig. 16 Fig. 17. Fig. 18. Fig. 19.

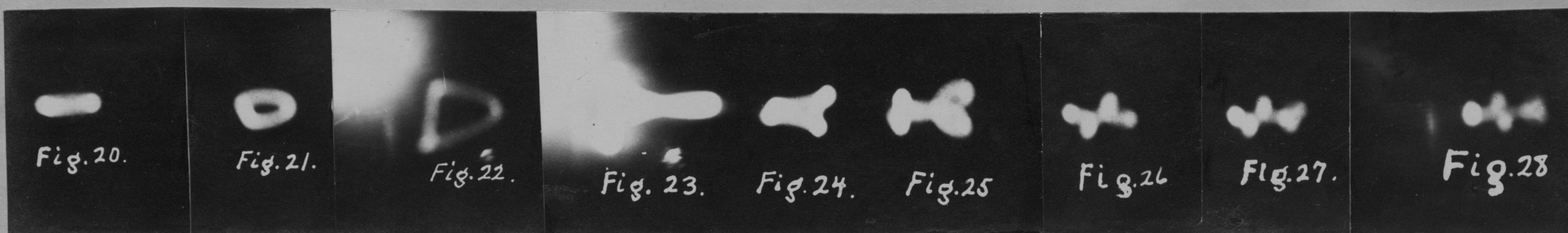


Fig. 20. Fig. 21. Fig. 22. Fig. 23. Fig. 24. Fig. 25 Fig. 26 Fig. 27. Fig. 28

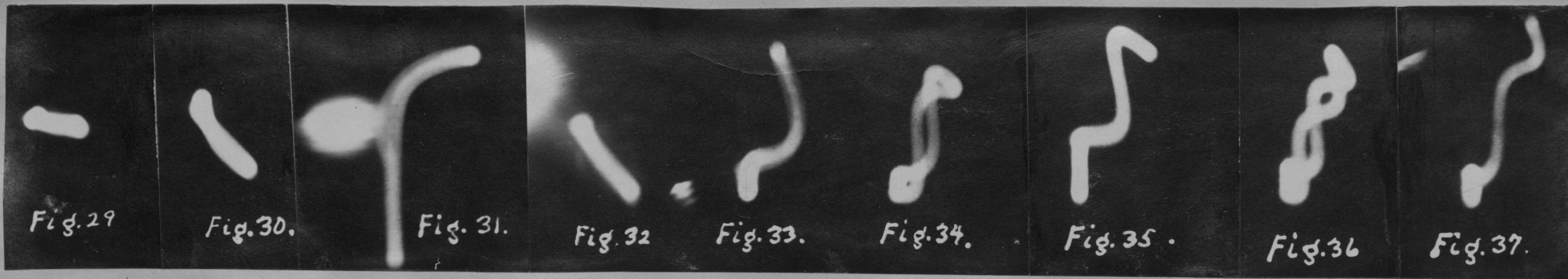


Fig. 29 Fig. 30. Fig. 31. Fig. 32 Fig. 33. Fig. 34. Fig. 35. Fig. 36 Fig. 37.

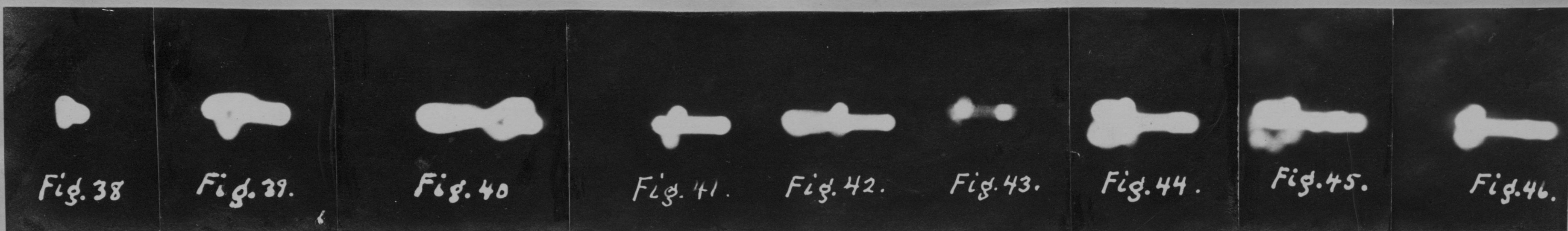


Fig. 38 Fig. 39. Fig. 40 Fig. 41. Fig. 42. Fig. 43. Fig. 44. Fig. 45. Fig. 46.

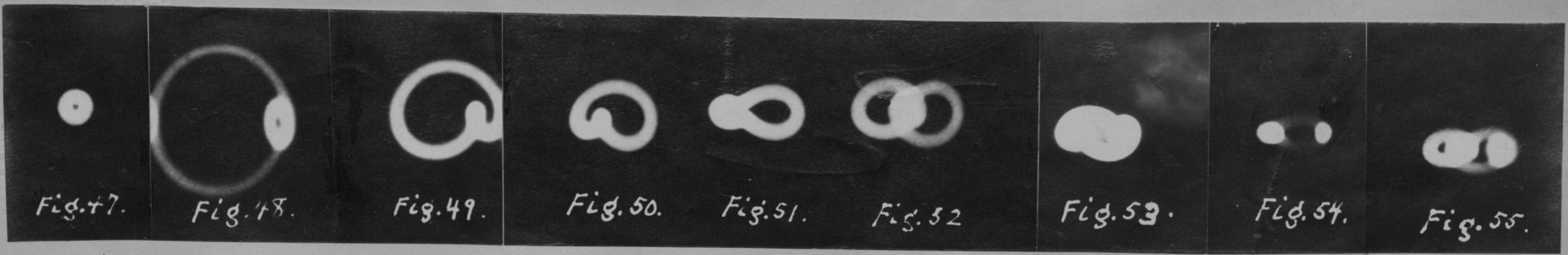


Fig. 47. Fig. 48. Fig. 49. Fig. 50. Fig. 51. Fig. 52 Fig. 53. Fig. 54. Fig. 55.

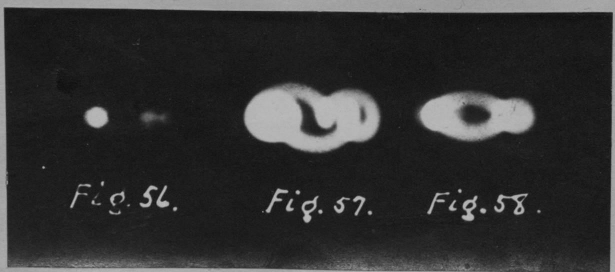


Fig. 56. Fig. 57. Fig. 58.